8. Součin a podíl komplexních čísel v goniometrickém tvaru

- předp. $z_1 = |z_1|(\cos\varphi_1 + i\sin\varphi_1)$ $\quad z_2 = |z_2|(\cos\varphi_2 + i\sin\varphi_2)$ $\quad [!\, z_1 \neq 0,\ z_2 \neq 0]$

- SOUČIN z_1, z_2: komplexní číslo $z = |z|(\cos\varphi + i\sin\varphi)$

$$z = z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot [\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

$\left[\begin{array}{ll} |z| = |z_1| \cdot |z_2| & \text{abs. hodn. VYNÁSOBÍME} \\ \varphi = \varphi_1 + \varphi_2 & \text{úhly SEČTEME} \end{array}\right]$

- PODÍL z_1, z_2: komplexní číslo $z = |z|(\cos\varphi + i\sin\varphi)$

$$z = \frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)]$$

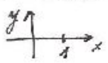
$\left[\begin{array}{ll} |z| = \frac{|z_1|}{|z_2|} & \text{abs. hodnoty VYDĚLÍME} \\ \varphi = \varphi_1 - \varphi_2 & \text{úhly ODEČTEME} \end{array}\right]$

- PLATÍ: $\dfrac{1}{\cos\varphi + i\sin\varphi} = \cos\varphi - i\sin\varphi$

důkaz: $\dfrac{1}{\cos\varphi + i\sin\varphi} = \dfrac{\overbrace{\cos 0 + i\sin 0}^{\text{FIGL}}}{\cos\varphi + i\sin\varphi} = \cos(0-\varphi) + i\sin(0-\varphi) = \overbrace{\cos(-\varphi) + i\sin(-\varphi)}^{\text{goniom. tvar}}$

$= \underbrace{\cos\varphi - i\sin\varphi}_{\text{(není gon. tvar)}}$ cbd. $\left[\begin{array}{ll} \cos(-\varphi) = \cos\varphi & \text{SUDÁ FCE} \\ \sin(-\varphi) = -\sin\varphi & \text{LICHÁ FCE} \end{array}\right]$

$\left[1 = \overbrace{\cos 0 + i\sin 0}^{\text{gon. tvar}} \quad |1| = 1,\ \varphi = 0 \right]$



Příklady

① Vypočítejte $z_1 \cdot z_2$, $\frac{z_1}{z_2}$

a) $z_1 = 3(\cos 45° + i\sin 45°)$

$z_2 = 4(\cos 75° + i\sin 75°)$

$z = z_1 z_2 = 3 \cdot 4\,[\cos(45° + 75°) + i\sin(45° + 75°)]$

$= 12\,(\overbrace{\cos 120° + i\sin 120°}^{\text{ je gon. tvar}})$

$\left[\ominus \quad \text{II. kv.} \quad \oplus \quad \varphi' = 60° \right]$

$= 12\,(\overbrace{-\underbrace{\cos 60°}_{\frac{1}{2}} + i\underbrace{\sin 60°}_{\frac{\sqrt{3}}{2}}}^{\text{není gon. tvar}})$

$= 12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -6 + 6\sqrt{3}i$

$z = \frac{z_1}{z_2} = \frac{3}{4}[\cos(45° - 75°) + i\sin(45° - 75°)]$

$= \frac{3}{4}\,(\overbrace{\cos(-30°) + i\sin(-30°)}^{\text{je gon. tvar}}) =$ $\quad$ +1.360° $\quad$ +1.360°

S VYUŽ. PERIODICNOSTI

$= \frac{3}{4}[\overbrace{\cos 330° + i\sin 330°}^{\text{je gon. tvar}}] =$

$\oplus \quad \text{IV. kv.} \quad \ominus \quad \varphi' = 30°$

S VYUŽ. PARITY

$= \frac{3}{4}\,(\overbrace{\underbrace{\cos 30°}_{\frac{\sqrt{3}}{2}} - i\underbrace{\sin 30°}_{\frac{1}{2}}}^{\text{není gon. tvar}}) =$

$= \frac{3}{4}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{3\sqrt{3}}{8} - \frac{3}{8}i$

b) $z_1 = 2\sqrt{2}\,(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi)$ $\quad z_2 = \sqrt{2}\,(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$

$z = z_1 z_2 = 2\sqrt{2} \cdot \sqrt{2}\left[\cos\left(\frac{\pi}{4} + \frac{7\pi}{4}\right) + i\sin\left(\frac{\pi}{4} + \frac{7\pi}{4}\right)\right] = 2 \cdot 2\,(\underbrace{\cos 2\pi}_{-1.2\pi} + i\underbrace{\sin 2\pi}_{-1.2\pi}) =$

$= 4(\cos 0 + i\sin 0) = 4(1 + 0i) = 4$

$z = \frac{z_1}{z_2} = \dfrac{2\sqrt{2}\,(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})}{\sqrt{2}\,(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})} = 2\left[\cos\left(\frac{\pi}{4} - \frac{7\pi}{4}\right) + i\sin\left(\frac{\pi}{4} - \frac{7\pi}{4}\right)\right] =$

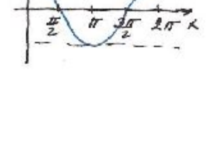
s využ. PERIODIC.

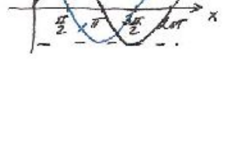
$= 2\left[\cos\left(\overset{+2\pi}{-\frac{3\pi}{2}}\right) + i\sin\left(\overset{+2\pi}{-\frac{3\pi}{2}}\right)\right] = 2\,(\underbrace{\cos\frac{\pi}{2}}_{0} + i\underbrace{\sin\frac{\pi}{2}}_{1}) = 2(0 + i) = 2i$

[s využ. PARITY $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$

$= 2\,(\underbrace{\cos\frac{3\pi}{2}}_{0} - i\underbrace{\sin\frac{3\pi}{2}}_{-1}) = 2(0 - (-1)i) =$

$= 2(0 + i) = 2i$]





② Vyjádřete v goniometrickém tvaru a potom v algebraickém

a) $\dfrac{1}{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}} = \dfrac{\cos 0 + i\sin 0}{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}} = \underbrace{\cos\left(\overset{+1\cdot 2\pi}{-\frac{\pi}{6}}\right)+i\sin\left(\overset{+2\pi}{-\frac{\pi}{6}}\right)}_{\text{GONIOM. TVAR 1}} = \underbrace{\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}}_{\text{GONIOM. TVAR 2}} =$

$\left[\begin{array}{l} 1 = \cos 0 + i\sin 0 \\ \varphi = 0 \\ |z| = |1| = 1 \end{array}\right]$

$= \cos 330° + i\sin 330° = \cos 30° - i\sin 30° = \underbrace{\frac{\sqrt{3}}{2} - \frac{1}{2}i}_{\text{ALG. TVAR}}$

$\left[\begin{array}{l} \oplus \quad \text{IV. kv.} \quad \ominus \\ \varphi' = 30° \quad (330° = 360° - \varphi') \\ \varphi' = \frac{\pi}{6} \end{array}\right] \quad \left[\overset{\text{nejsou gonio. tvary}}{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}\right]$

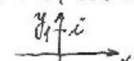
– na alg. tvar lze z gon. tvaru 1 s využ. $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$

$\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right) = \underbrace{\cos\overset{30°}{\frac{\pi}{6}} - i\sin\overset{30°}{\frac{\pi}{6}}}_{\text{není gon. tvar}} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

b) DÚ $\dfrac{i}{\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi} = \dfrac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

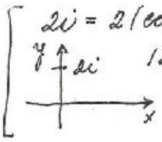
$\left[\frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \frac{\pi}{6}\right]$

$\left[\begin{array}{l} i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \\ |i| = 1 \\ \varphi = \frac{\pi}{2} \end{array}\right]$



c) $\dfrac{\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}}{2i} = \dfrac{\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}}{2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)} = \frac{1}{2}\left[\cos\left(\frac{5\pi}{6}-\frac{\pi}{2}\right) + i\sin\left(\frac{5\pi}{6}-\frac{\pi}{2}\right)\right] =$

$\left[\begin{array}{l} 2i = 2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right) \\ |2i| = 2 \\ \varphi = 90° = \frac{\pi}{2} \end{array}\right]$



$= \frac{1}{2}\left(\underbrace{\cos\overset{60°}{\frac{\pi}{3}}}_{\frac{1}{2}} + i\underbrace{\sin\overset{60°}{\frac{\pi}{3}}}_{\frac{\sqrt{3}}{2}}\right) = \frac{1}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{1}{4} + \frac{\sqrt{3}}{4}i$

d) DÚ $\dfrac{1}{i} = \dfrac{\cos 0 + i\sin 0}{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}} = \cos\left(\overset{+2\pi}{-\frac{\pi}{2}}\right) + i\sin\left(\overset{+2\pi}{-\frac{\pi}{2}}\right) = \underbrace{\cos\frac{3\pi}{2}}_{0} + i\underbrace{\sin\frac{3\pi}{2}}_{-1 \text{ z grafu}} = 0 - 1i = -i$

↳ s využ. PARITY

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = 0 - 1i = -i$

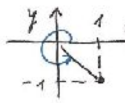
$\left[\begin{array}{l} 1 = \cos 0 + i\sin 0 \\ i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \end{array}\right]$ nebo

$\frac{1}{i} = \frac{1}{i}\cdot\frac{-i}{-i} = \underset{\text{alg.}}{-i} = \underbrace{\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}}_{\text{gon. tvar}}$

③ Vyjádřete v goniometrickém tvaru

a) $(1-i)\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) =$

$\left[\begin{array}{l} 1 - i = [1, -1] \\ |z| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \\ \varphi = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \end{array}\right]$



$= \sqrt{2}\left[\cos\left(\frac{7\pi}{4} + \frac{5\pi}{12}\right) + i\sin\left(\frac{7\pi}{4} + \frac{5\pi}{12}\right)\right]$

$= \sqrt{2}\left[\cos\frac{26\pi}{12} + i\sin\frac{26\pi}{12}\right] = \qquad \frac{7\pi}{4} = \frac{21\pi}{12}$

$= \sqrt{2}\left(\cos\frac{13\pi}{6} + i\sin\frac{13\pi}{6}\right) = \sqrt{2}\left(\underbrace{\cos\overset{30°}{\frac{\pi}{6}}}_{\frac{\sqrt{3}}{2}} + i\underbrace{\sin\overset{30°}{\frac{\pi}{6}}}_{\frac{1}{2}}\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

$\left[\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}\right]$

NEBO

$\left[1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right] \ldots \sqrt{2}\left[\cos\left(-\frac{\pi}{4} + \frac{5\pi}{12}\right) + i\sin\left(-\frac{\pi}{4} + \frac{5\pi}{12}\right)\right] =$

$= \sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

$\left[\frac{5\pi}{12} - \frac{\pi}{4} = \frac{2\pi}{12} = \frac{\pi}{6}\right]$

MOŽNÁ JEDNODUŠŠÍ – PRO NĚKOHO

b) $(\cos \frac{2}{5}\pi + i \sin \frac{2}{5}\pi)\, i = (\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5})(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) =$

DÚ

$= \cos \frac{9\pi}{10} + i \sin \frac{9\pi}{10} =$ $\left[\frac{2\pi}{5} + \frac{\pi}{2} = \frac{4\pi + 5\pi}{10} = \frac{9\pi}{10}\right]$

$= \underbrace{\cos 324° + i \sin 324°}_{\text{na kalkulačce} \Rightarrow \text{alg. tvar (zaokrouhlit)}} = 0{,}8 + (-0{,}6)i =$ $\left[\frac{9\pi}{10} = \frac{9 \cdot 360°}{10} = 9 \cdot 36° = 324°\right]$

$= 0{,}8 - 0{,}6i$ $[0{,}8090 - 0{,}5878i]$

c) $2i \sin \frac{1}{4}\pi \left(\cos \frac{7\pi}{3} + i \sin \frac{7\pi}{3}\right)\left(\cos(-\frac{2\pi}{3}) + i \sin(-\frac{2\pi}{3})\right) =$

DÚ $= 2 \cdot \underbrace{\frac{\sqrt{2}}{2}(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})}_{i \text{ v gon. tvaru}} (\cos \frac{7\pi}{3} + i \sin \frac{7\pi}{3})(\cos(-\frac{2\pi}{3}) + i \sin(-\frac{2\pi}{3})) =$

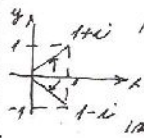
$= \sqrt{2}\left[\cos(\frac{\pi}{2} + \frac{7\pi}{3} - \frac{2\pi}{3}) + i \sin(\frac{\pi}{2} + \frac{7\pi}{3} - \frac{2\pi}{3})\right]$

$= \sqrt{2}\left(\cos \frac{13\pi}{6} + i \sin \frac{13\pi}{6}\right) = \sqrt{2}\left(\cos \overset{30°}{\frac{\pi}{6}} + i \sin \overset{30°}{\frac{\pi}{6}}\right)$ $\left[\frac{13\pi}{6} = 2\frac{1}{6}\pi = \frac{1}{6}\pi\right]$

$= \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

d) $\frac{\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}}{1 - i} \cdot \frac{1 + i}{\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}} =$

DÚ



$|x| = \sqrt{1+1} = \sqrt{2}$, $\varphi = 45° = \frac{\pi}{4}$

$\varphi = -45° = -\frac{\pi}{4}$, $|x| = \sqrt{1 + (-1)^2} = \sqrt{2}$

$= \frac{\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}}{\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}} \cdot \frac{\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})}{\sqrt{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))} =$

$= \left[\cos\left(\underbrace{\frac{5\pi}{6} - \frac{7\pi}{6}}_{-\frac{2\pi}{6} = -\frac{\pi}{3}}\right) + i \sin\left(\frac{5\pi}{6} - \frac{7\pi}{6}\right)\right] \cdot \left[\cos\left(\frac{\pi}{4} - (-\frac{\pi}{4})\right) + i \sin\left(\frac{\pi}{4} - (-\frac{\pi}{4})\right)\right] =$

$= \left[\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3})\right] \cdot \left[\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right] = \left[\cos(-\frac{\pi}{3} + \frac{\pi}{2}) + i \sin(-\frac{\pi}{3} + \frac{\pi}{2})\right] =$

$= \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$

(4) Zapište v goniometrickém tvaru

a) $\frac{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}{\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}} = \frac{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}{\cos(-\frac{\pi}{6}) + i \sin(-\frac{\pi}{6})} = \cos\left(\frac{\pi}{3} - (-\frac{\pi}{6})\right) + i \sin\left(\frac{\pi}{3} - (-\frac{\pi}{6})\right) =$

! není gon. tvar ! je gon. tvar

$\left[\cos \frac{\pi}{6} = \cos(-\frac{\pi}{6}) \quad -\sin \frac{\pi}{6} = \sin(-\frac{\pi}{6})\right]$ využ. PARITU $= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i$

b) $\frac{i}{\sin\varphi + i\cos\varphi} = \frac{i}{\sin\varphi + i\cos\varphi} \cdot \frac{i}{i} = \frac{i^2}{i\sin\varphi + i^2\cos\varphi} = \frac{-1}{i\sin\varphi - \cos\varphi} =$

DÚ není gon. tvar FIGL

$= \frac{-1}{-1(\cos\varphi - i\sin\varphi)} = \frac{1}{\cos\varphi - i\sin\varphi} = \frac{1}{\cos(-\varphi) + i\sin(-\varphi)} = \frac{\cos 0 + i \sin 0}{\cos(-\varphi) + i\sin(-\varphi)} =$

využ. paritu je gon. tvar

$= \cos\varphi + i\sin\varphi$

⑤ Užitím vztahu pro násobení a dělení komplexních čísel v goniometrickém tvaru dokažte:

a) $\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta$

$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \sin\beta$

zvolíme komplexní $\underset{|x|=1}{\text{jednotky}}$: $a = \cos\alpha + i\sin\alpha$ $\quad b = \cos\beta + i\sin\beta$

1. určíme $a \cdot b$ podle vztahu pro násobení kompl. čísel v gon. tvaru

$$ab = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = \underbrace{\cos(\alpha+\beta)}_{Re} + i\underbrace{\sin(\alpha+\beta)}_{Im}$$

2. určíme $a \cdot b$ podle vztahu pro násobení dvojčlenů (kompl. č. v alg. tvaru)

$$ab = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) =$$

$$= \cos\alpha \cdot \cos\beta + i\cos\alpha\sin\beta + i\sin\alpha\cos\beta + \underbrace{i^2\sin\alpha\sin\beta}_{-\sin\alpha\sin\beta} =$$

$$= \underbrace{\cos\alpha\cos\beta - \sin\alpha\sin\beta}_{Re} + i(\underbrace{\cos\alpha\sin\beta + \sin\alpha\cos\beta}_{Im})$$

3. porovnáme reálné části a imag. části získané oběma způsoby

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ cbd.

b) $\cos(\alpha-\beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta$

3b $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

zvolíme kompl. jednotky : $a = \cos\alpha + i\sin\alpha$ $\quad b = \cos\beta + i\sin\beta$

1. určíme $\frac{a}{b}$ podle vztahu pro dělení kompl. čísel v goniom. tvaru

$$\frac{a}{b} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \underbrace{\cos(\alpha-\beta)}_{Re} + i\underbrace{\sin(\alpha-\beta)}_{Im}$$

2. určíme $\frac{a}{b}$ podle dělení v alg. tvaru

$$\frac{a}{b} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} \cdot \frac{\cos\beta - i\sin\beta}{\cos\beta - i\sin\beta} =$$

$$= \frac{\cos\alpha\cos\beta - i\cos\alpha\sin\beta + i\sin\alpha\cos\beta - \overbrace{i^2\sin\alpha\sin\beta}^{+\sin\alpha\sin\beta}}{\cos^2\beta - \underbrace{i^2\sin^2\beta}_{+\sin^2\beta}}$$

$$= \frac{\sin\alpha\sin\beta + \cos\alpha\cos\beta + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\underbrace{\sin^2\beta + \cos^2\beta}_{1}}$$

$$= \underbrace{\sin\alpha\sin\beta + \cos\alpha\cos\beta}_{Re} + i(\underbrace{\sin\alpha\cos\beta - \cos\alpha\sin\beta}_{Im})$$

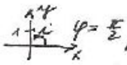
3. porovnáme Re, Im získané oběma způsoby $\quad$ $[\sin^2 x + \cos^2 x = 1]$

$\cos(\alpha-\beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\quad$ cbd.

⑥ Urči argument tak, aby platila rovnost

2.34 a) $\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)(\cos\varphi+i\sin\varphi)=i$ $\qquad \left[\, i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2} \quad |i|=1 \,\right]$



3

$$\cos\left(\tfrac{2\pi}{3}+\varphi\right)+i\sin\left(\tfrac{2\pi}{3}+\varphi\right)=\cos\tfrac{\pi}{2}+i\sin\tfrac{\pi}{2}$$

z rovnosti kompl. č. v goniom. tvaru

$$\tfrac{2\pi}{3}+\varphi=\tfrac{\pi}{2}+2k\pi \qquad \text{(periodicita)}$$

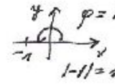
$$\varphi=\tfrac{\pi}{2}-\tfrac{2\pi}{3}+2k\pi$$

$$\varphi=\tfrac{3\pi-4\pi}{6}+2k\pi$$

$$\varphi=-\tfrac{\pi}{6}+2k\pi$$

$$\left(\varphi=\tfrac{11\pi}{6}+2k\pi\right)$$

b) $\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)(\cos\varphi+i\sin\varphi)=-1$ $\qquad \left[\, -1=\cos\pi+i\sin\pi \,\right]$



3b

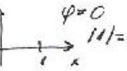
$$\cos\left(\tfrac{7\pi}{4}+\varphi\right)+i\sin\left(\tfrac{7\pi}{4}+\varphi\right)=\cos\pi+i\sin\pi$$

z rovnosti kompl. č. v goniom. tvaru

$$\tfrac{7\pi}{4}+\varphi=\pi+2k\pi$$

$$\varphi=\pi-\tfrac{7\pi}{4}+2k\pi=-\tfrac{3\pi}{4}+2k\pi$$

c) $\dfrac{\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}}{\cos\varphi+i\sin\varphi}=1$ $\qquad \left[\, 1=\cos 0+i\sin 0 \,\right]$



3(DU)

$$\cos\left(\tfrac{7\pi}{6}-\varphi\right)+i\sin\left(\tfrac{7\pi}{6}-\varphi\right)=\cos 0+i\sin 0$$

rovnost kompl. č. v gon. tvaru

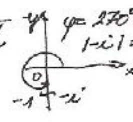
$$\tfrac{7\pi}{6}-\varphi=0+2k\pi$$

$$\tfrac{7\pi}{6}-2k\pi=\varphi \qquad k\in\mathbb{Z}$$

přeznačíme $(-k\to k)$

$$\varphi=\tfrac{7\pi}{6}+2k\pi$$

d) $\dfrac{\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}{\cos\varphi+i\sin\varphi}=-i$ $\qquad \left[\, -i=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2} \,\right]$



3b

$$\cos\left(\tfrac{\pi}{8}-\varphi\right)+i\sin\left(\tfrac{\pi}{8}-\varphi\right)=\cos\tfrac{3\pi}{2}+i\sin\tfrac{3\pi}{2}$$

rovnost kompl. č. v gon. tvaru

$$\tfrac{\pi}{8}-\varphi=\tfrac{3\pi}{2}+2k\pi \qquad (-k\to k)$$

$$\varphi=\tfrac{\pi}{8}-\tfrac{3\pi}{2}+2k\pi$$

$$\varphi=-\tfrac{11\pi}{8}+2k\pi \qquad k\in\mathbb{Z}$$